day 85

FRQ 5)

V is changing at a rate of $-5\pi\sqrt{h}$ $\frac{\text{cu in}}{\text{sec}}$

a) show that $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$

i) eqn always true: $V = \pi r^2 h = 25\pi h$

ii) derivative eqn: $\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

iii) given: $\frac{dV}{dt} = -5\pi\sqrt{h}$ so

$$\frac{-5\pi\sqrt{h}}{25\pi} = \frac{25\pi}{25\pi}\frac{dh}{dt}$$

and $\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$ which is what I was supposed to show

day 85

5b) Given that $h=17$ at $t=0$

Solve $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a fn of t

IMPORTANT
ALWAYS a 5- or 6-point question on AP (out of 108)

1) separate variables $\frac{dh}{\sqrt{h}} = -\frac{1}{5}dt$

2) anti derivatives of both sides $\int \frac{dh}{\sqrt{h}} = -\frac{1}{5}\int dt$

$h^{-\frac{1}{2}}$

$2\sqrt{h} = -\frac{t}{5} + C$

include +C
no more pts if you forget

3) solve initial condition

when $t=0, h=17$, so

$2\sqrt{17} = -\frac{0}{5} + C$

$\therefore C = 2\sqrt{17}$

4) rewrite eqn

$2\sqrt{h} = -\frac{t}{5} + 2\sqrt{17}$

$\sqrt{h} = -\frac{t}{10} + \sqrt{17}$

solve for h

$h = \left(-\frac{t}{10} + \sqrt{17}\right)^2$ ✓

day 85

c) $h = \left(-\frac{t}{10} + \sqrt{17}\right)^2$ ✓

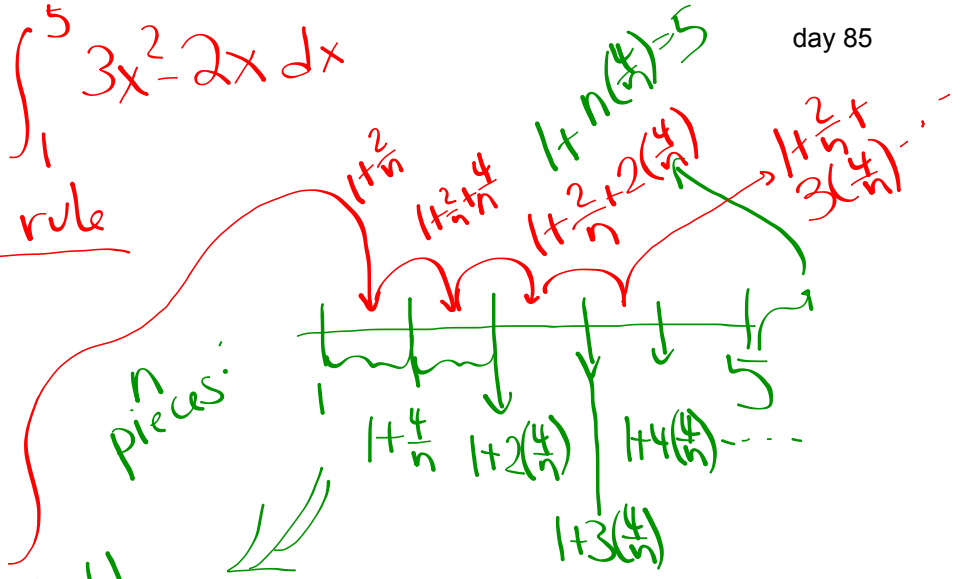
When is coffeepot empty? When $h = 0$!

$0 = \left(-\frac{t}{10} + \sqrt{17}\right)^2$

$\Rightarrow 0 = -\frac{t}{10} + \sqrt{17} \Rightarrow \sqrt{17} = \frac{t}{10} \Rightarrow t = 10\sqrt{17}$ seconds

5.7/21 $\int_1^5 3x^2 - 2x\,dx$ day 85

midpt rule



midpt (ugh)

midpt. approx. $A \approx \left(\frac{4}{n}\right)\left(f\left(1+\frac{2}{n}+0\left(\frac{4}{n}\right)\right)+\left(\frac{4}{n}\right)\left(1+\frac{2}{n}+1\left(\frac{4}{n}\right)\right)\right.$

W # $\left.+\left(\frac{4}{n}\right)\left(1+\frac{2}{n}+2\left(\frac{4}{n}\right)\right)+\cdots\right.$

changes

$$A \approx \sum_{K=0}^{(n-1)} \left(\frac{4}{n}\right) f\left(1+\frac{2}{n}+K\left(\frac{4}{n}\right)\right)$$

$$= \sum_{K=0}^{n-1} \left(\frac{4}{n}\right)\left(3\left(1+\frac{2}{n}+K\frac{4}{n}\right)^2 - 2\left(1+\frac{2}{n}+K\left(\frac{4}{n}\right)\right)\right.$$

↓ CALCULATOR

→ 32 STO→ (ALPHA N)

→ sum(seq((4÷N)*(3(1+2÷N+X·4÷N)^2−2 ⋯ :),

64 STO→(ALPHA N)

X,0,N−1)